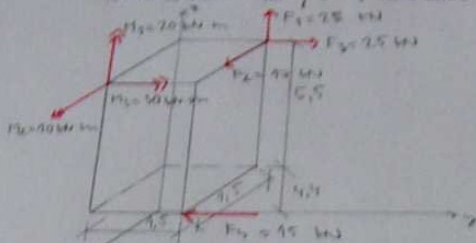


Pr. 9 TUHÉ TELESO JE ZATAŽENÉ NĚKOLKOU SIL F_i A SILOVÝMI
 DVUHCAMI M_i . VZHLÁDOM K BODU O URČÍME VÝSLEDNÝ
 BINĚKTOR R , M . NAKRESLETE OBR. R , M .



$$F_{Rx} = \sum_{i=1}^3 F_{ix} \quad F_{1x} = F_2 = 17 \text{ kN}$$

$$F_{Ry} = \sum_{i=1}^3 F_{iy} \quad F_{1y} = F_3 - F_1 = 25 - 15 = 10 \text{ kN}$$

$$F_{Rz} = \sum_{i=1}^3 F_{iz} \quad F_{1z} = F_1 = 25 \text{ kN}$$

$$F_R = \sqrt{F_{Rx}^2 + F_{Ry}^2 + F_{Rz}^2} = \sqrt{17^2 + 10^2 + 25^2} = 31,843 \text{ kN}$$

$$\cos \alpha_R = \frac{F_{Rx}}{F_R} = \frac{17}{31,843} = 0,53386 \Rightarrow \alpha_R = 57,732^\circ$$

$$\cos \beta_R = \frac{F_{Ry}}{F_R} = \frac{10}{31,843} = 0,31404 \Rightarrow \beta_R = 71,697^\circ$$

$$\cos \gamma_R = \frac{F_{Rz}}{F_R} = \frac{25}{31,843} = 0,78510 \Rightarrow \gamma_R = 38,269^\circ$$

KONTROLA

$$\cos^2 \alpha_R + \cos^2 \beta_R + \cos^2 \gamma_R = 1$$

$$0,53386^2 + 0,31404^2 + 0,78510^2 = 1$$

$$1,000009631 = 1 \quad \checkmark$$

$$M_x = \sum_{i=1}^3 M_{ix} + \sum_{i=1}^3 M_x = F_1 \cdot 1,5 - F_3 \cdot 9,9 + F_2 \cdot 1,5 + M_2 =$$

$$= 25 \cdot 1,5 - 25 \cdot 9,9 + 15 \cdot 1,5 + 10 = -177,5 \text{ kN}\cdot\text{m}$$

$$M_y = \sum_{i=1}^3 M_{iy} + \sum_{i=1}^3 M_y = F_2 \cdot 9,9 + M_3 =$$

$$= 17 \cdot 9,9 + 30 = 198,3 \text{ kN}\cdot\text{m}$$

$$M_z = \sum_{i=1}^3 M_{iz} + \sum_{i=1}^3 M_z = -F_2 \cdot 1,5 - F_1 \cdot 1,5 + M_1 =$$

$$= -17 \cdot 1,5 - 15 \cdot 1,5 + 20 = -28 \text{ kN}\cdot\text{m}$$

$$M = \sqrt{M_x^2 + M_y^2 + M_z^2} = \sqrt{-177,5^2 + 198,3^2 + (-28)^2} = 267,606 \text{ kN}\cdot\text{m}$$

$$\cos \lambda = \frac{M_x}{M} = \frac{-177,5}{267,606} = -0,66328 \Rightarrow \lambda = 131,551^\circ$$

$$\cos \mu = \frac{M_y}{M} = \frac{198,3}{267,606} = 0,74107 \Rightarrow \mu = 42,182^\circ$$

$$\cos \nu = \frac{M_z}{M} = \frac{-28}{267,606} = -0,10463 \Rightarrow \nu = 96,005^\circ$$

KONTROLA

$$\cos^2 \alpha + \cos^2 \mu + \cos^2 \beta = 1$$

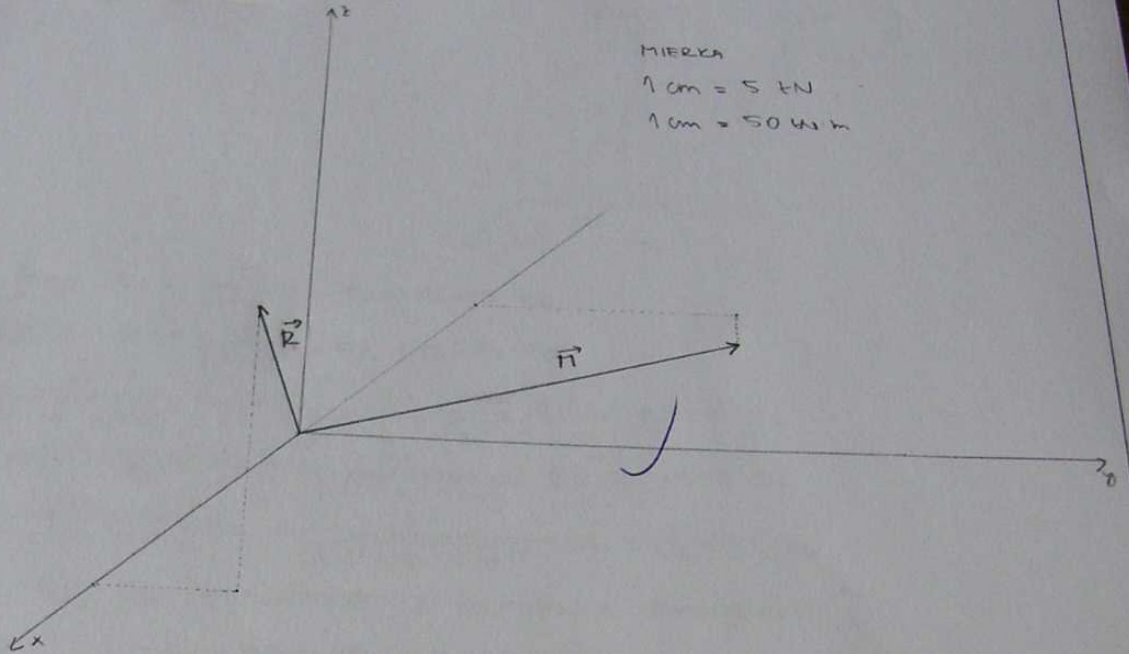
$$-0,66308^2 + 0,74101^2 + (-0,10462)^2 = 1$$

$$0,99998 \approx 1$$

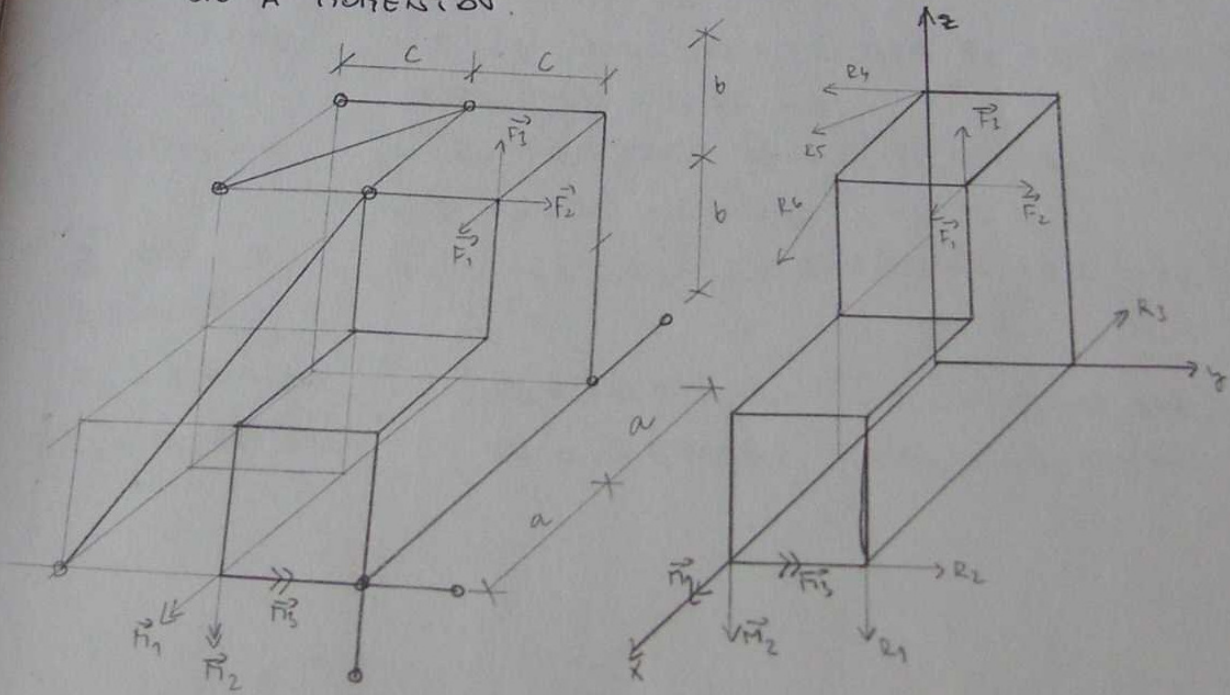
MIERKA

$$1 \text{ cm} = 5 \text{ tN}$$

$$1 \text{ cm} = 50 \text{ kNm}$$

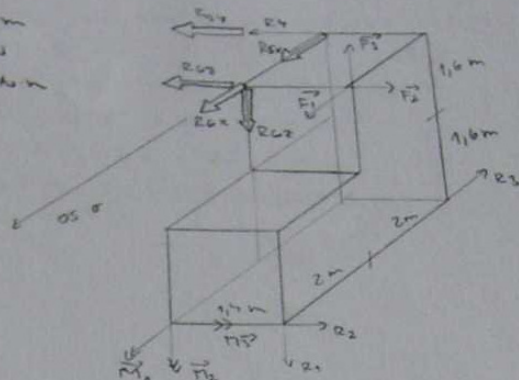


PR.10 URČITE REAKCIE NA TUHOM TELESE, ZATÁŽENOM SÚSTAVOU SIL A MOMENTOV.



DANE:

$a = 2 \text{ m}$, $b = 3 \text{ m}$, $c = 1,5 \text{ m}$
 $F_1 = 5 \text{ kN}$, $F_2 = 8 \text{ kN}$, $F_3 = 5 \text{ kN}$
 $M_1 = 2 \text{ kNm}$, $M_2 = 3 \text{ kNm}$, $M_3 = 6 \text{ kNm}$



$$R_{5x} = \frac{a}{\sqrt{a^2+c^2}} \cdot R_5 = \frac{2}{\sqrt{2^2+1,5^2}} \cdot R_5 = 0,819 R_5$$

$$R_{5y} = \frac{c}{\sqrt{a^2+c^2}} \cdot R_5 = \frac{1,5}{\sqrt{2^2+1,5^2}} \cdot R_5 = 0,574 R_5$$

$$R_{6x} = \frac{a}{\sqrt{a^2+c^2+(2b)^2}} \cdot R_6 = \frac{2}{\sqrt{2^2+1,5^2+3,2^2}} \cdot R_6 = 0,497 R_6$$

$$R_{6y} = \frac{c}{\sqrt{a^2+c^2+(2b)^2}} \cdot R_6 = \frac{1,5}{\sqrt{2^2+1,5^2+3,2^2}} \cdot R_6 = 0,378 R_6$$

$$R_{6z} = \frac{2b}{\sqrt{a^2+c^2+(2b)^2}} \cdot R_6 = \frac{3,2}{\sqrt{2^2+1,5^2+3,2^2}} \cdot R_6 = 0,795 R_6$$

REAKCIE $R_1 - R_6$ VYPOČÍTAME Z PODMIENOK ROVNOVÁHY:

$$\sum_{i=1}^n F_{ix} = 0$$

$$-R_3 + 0,819 R_5 + 0,497 R_6 + 3 = 0$$

$$\sum_{i=1}^n F_{iy} = 0$$

$$R_2 - R_4 - 0,574 R_5 - 0,378 R_6 + 8 = 0$$

$$\sum_{i=1}^n F_{iz} = 0$$

$$-R_1 - 0,795 R_6 + 5 = 0$$

$$\sum_{i=1}^n M_{ix} = 0$$

$$-R_1 \cdot 1,5 + 3,2 \cdot R_5 + 3,2 \cdot 0,574 \cdot R_5 + 3,2 \cdot 0,378 \cdot R_6 + 2 + 5 \cdot 1,5 - 8 \cdot 3,2 = 0$$

$$\sum_{i=1}^n M_{iy} = 0$$

$$4 \cdot R_1 + 3,2 \cdot 0,819 R_5 + R_6 (0,497 \cdot 3,2 + 0,795 \cdot 2) + 6 + 3 \cdot 3,2 - 5 \cdot 2 = 0$$

$$\sum_{i=1}^n M_{iz} = 0$$

$$4 \cdot R_2 + 1,5 R_3 - 2 \cdot 0,248 R_6 - 4 + 8 \cdot 2 - 3 \cdot 1,5 = 0$$

RIEŠENIE

$$R_1 = 5,971 \text{ kN}$$

$$R_2 = -0,199 \text{ kN}$$

$$R_3 = -5,607 \text{ kN}$$

$$R_4 = 13,827 \text{ kN}$$

$$R_5 = -9,768 \text{ kN}$$

$$R_6 = -1,222 \text{ kN}$$

KONTROLA

RIEŠENIA :

$$\sum F_x = 0$$

$$-R_3 + 0,1819 R_5 + 0,4997 R_6 + 3 = 51007 - 0,1819 \cdot 91,768 - 0,4997 \cdot 11,222 + 3 = 0$$

$$\sum F_y = 0$$

$$R_2 - R_5 - 0,5774 R_5 - 0,348 R_6 + 8 = -0,1999 \cdot 15,827 + 0,5774 \cdot 9,768 + 0,348 \cdot 11,222 + 8 = 0$$

$$\sum_{i=1}^n M_{i0} = 0$$

$$F_3 \cdot 4 + M_1 + R_2 \cdot 26 - R_1 \cdot 6 = 0$$

$$5 \cdot 115 + 2 - 0,1999 \cdot 312 - 51,971 \cdot 115 = 0$$